Paracatadioptric Camera Calibration Using Lines

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Abstract

Paracatadioptric sensors combine a parabolic shaped mirror and a camera inducing an orthographic projection. Such a configuration provides a wide field of view while keeping a single effective viewpoint. Previous work in central catadioptric sensors proved that a line projects into a conic curve and that three line images are enough to calibrate the system. However the estimation of the conic curves where lines are mapped is hard to accomplish. In general only a small arc of the conic is visible in the image and conventional conic fitting techniques are unable to correctly estimate the curve. The present work shows that a set of conic curves corresponds to paracatadioptric line images if, and only if, certain properties are verified. These properties are used to constraint the search space and correctly estimate the curves. The accurate estimation of a minimum of three line images allows the complete calibration of the paracatadioptric camera. If the camera is skewless and the aspect ratio is known then the conic fitting problem is solved naturally by an eigensystem. For the general situation the conic curves are estimated using non-linear optimization.

1 Introduction

Omnidirectional vision is becoming an increasingly important sub-area in computer vision research. The approach of combining mirrors with conventional cameras to enhance sensor field of view is referred to as catadioptric image formation. Catadioptric sensors with an unique effective viewpoint are of primary interest. The entire class of catadioptric configurations verifying the fixed viewpoint constraint is derived in [1]. Panoramic central catadioptric systems can be built by combining an hyperbolic mirror with a perspective camera and, a parabolic mirror with an orthographic camera (paracatadioptric sensor). The construction of the former requires a careful alignment between the mirror and the imaging device. The camera projection center must be positioned in the outer focus of the hyperbolic reflective surface. The paracatadioptric camera is easier to construct being broadly used in vision applications.

In [2], Geyer and Daniilidis introduce for the first time an unifying theory for general central catadioptric image formation. A modified version of this mapping model is proposed in [3]. It is shown that central catadioptric projection is isomorphic to a projective mapping from a sphere, centered in the effective viewpoint, to a plane with a projection center on the perpendicular to the plane. For the particular case of paracatadioptric sensors the projection center lies on the sphere and the projective mapping is a stereographic projection. The plane and the final catadioptric image are related by a collineation depending on the mirror and camera intrinsic parameters. The system is calibrated when this collineation is known. It has already been proved that any central panoramic system can be fully calibrated from the image of three lines in general position [3]. However, since lines are mapped into conic curves which are only partially visible, the accurate estimation of catadioptric line images is far from being a trivial task [7].

The present paper focuses on paracatadioptric camera calibration using lines in general position. If it is true that any line maps into a conic in the catadioptric image plane, it is not true that any conic is the image of a line. We derive for the first time the necessary and sufficient conditions that must be verified by a set of conic curves to be the paracatadioptric projection of lines. We also show that the derived conditions can be used to accurately estimate the line images by non-linear optimization. Moreover if the system is skewless and the aspect ratio is known then the lines can be computed by solving an eigensystem. Given the image of at least three lines the paracatadioptric camera is easily calibrated using the results presented in [3].

2 Previous Work

The image formation model for paracatadioptric systems and the calibration algorithm using three or more lines are introduced. For further details please consult [3].

Consider a paracatadioptric system combining a parabolic mirror with latus rectum h, and an orthographic camera. The principal axis of the camera must be aligned with the symmetry axis of the paraboloid. The paracatadioptric projection can be modeled by a stereographic projection from an unitary sphere, centered in the effective

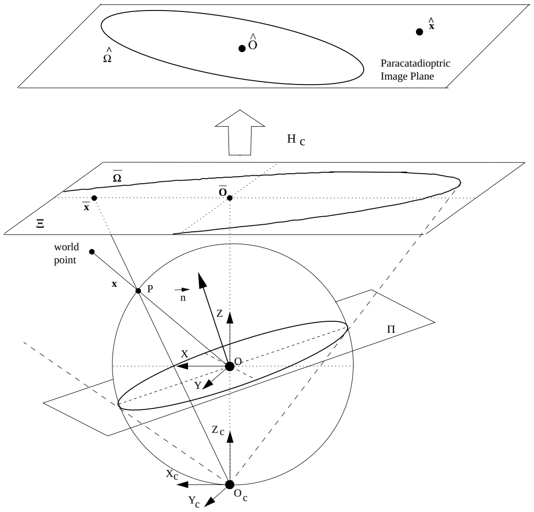

Figure 1: Model for paracatadioptric image formation

Step 1	Determine the catadioptric line images $\hat{\mathbf{\Omega}}_\mathbf{i}$ for $i = 1, 2, 3 \ldots K$
Step 2	For each pair of conics $\hat{\mathbf{\Omega}}_\mathbf{i}$, $\hat{\mathbf{\Omega}}_\mathbf{j}$, compute the intersection points $\hat{\mathbf{F}}_\mathbf{ij}$, $\hat{\mathbf{B}}_\mathbf{ij}$ and determine the corresponding line $\hat{\mu}_\mathbf{ij} = \hat{\mathbf{F}}_\mathbf{ij} \wedge \hat{\mathbf{B}}_\mathbf{ij}$
Step 3	Estimate the image center $\hat{\mathbf{O}}$ which is the intersection point of lines $\hat{\mu}_\mathbf{ij}$.
Step 4	For each conic $\hat{\mathbf{\Omega}}_\mathbf{i}$ compute the polar line $\hat{\pi}_\mathbf{i}$ of the image center $\hat{\mathbf{O}}$ ($i = 1, 2, 3 \ldots K$).
Step 5	For each conic curve obtain the points $\hat{\mathbf{I}}_\mathbf{i}$ and $\hat{\mathbf{J}}_\mathbf{i}$ where line $\hat{\pi}_\mathbf{i}$ intersects $\hat{\mathbf{\Omega}}_\mathbf{i}$ ($i = 1, 2, 3 \ldots K$)
Step 6	Estimate the conic $\hat{\mathbf{\Omega}}_\infty$ going through points $\hat{\mathbf{I}}_\mathbf{i}$, $\hat{\mathbf{J}}_\mathbf{i}$ ($i = 1, 2, 3 \ldots K$)
Step 7	Compute the Cholesky decomposition of $\hat{\mathbf{\Omega}}_\infty$ to derive matrix $\mathbf{H_c}$

Table 1: Calibrating a paracatadioptric system using K lines

viewpoint, into a plane $\mathbf{\Xi}$ as shown in Fig. 1.

The world point shown in Fig. 1 is imaged at point $\hat{\mathbf{x}}$ in the paracatadioptric image plane. To each visible scene point corresponds an oriented projective ray $\mathbf{x} = (x, y, z)^t$, joining the 3D point with the projection center $\mathbf{O}$. The projective ray intersects the unit sphere in a single point $\mathbf{P}$. Consider a point $\mathbf{O_c}$, with coordinates $(0, 0, -1)^t$, which lies in the unitary sphere. To each $\mathbf{x}$ corresponds an oriented projective ray $\bar{\mathbf{x}}$ joining $\mathbf{O_c}$ with the intersection point $\mathbf{P}$. The non-linear mapping $\mathbf{F}$ (equation 1) corresponds to projecting the scene in the unity sphere surface and then re-projecting the points on the sphere into a plane $\mathbf{\Xi}$ from a novel projection center $\mathbf{O_c}$. Points in catadioptric image plane $\hat{\mathbf{x}}$ are obtained after a collineation $\mathbf{H_c}$ of 2D projective points $\bar{\mathbf{x}}$. Equation 2 shows that $\mathbf{H_c}$ depends on the intrinsic parameters $\mathbf{K_c}$ of the orthographic camera, and on the latus rectum of the parabolic mirror.

$$\mathbf{F}(\mathbf{x}) = (x, y, z + \sqrt{x^2 + y^2 + z^2})^t \tag{1}$$

$$\hat{\mathbf{x}} = \underbrace{\mathbf{K_c} \begin{bmatrix} \frac{h}{2} & 0 & 0 \\ 0 & \frac{h}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{\mathbf{H_c}} \bar{\mathbf{x}} \tag{2}$$

Consider the plane $\mathbf{\Pi} = (\mathbf{n}, 0)^t$ going through the effective viewpoint $\mathbf{O}$ as depicted in Fig. 1 ($\mathbf{n} = (n_x, n_y, n_z)^t$). The paracatadioptric image of any line lying on $\mathbf{\Pi}$ is the conic curve $\hat{\mathbf{\Omega}}$. The line in the scene is projected into a great circle in the sphere surface. This great circle is the curve of intersection of plane $\mathbf{\Pi}$, containing both the line and the projection center $\mathbf{O}$, and the unit sphere. The projective rays $\bar{\mathbf{x}}$, joining $\mathbf{O_c}$ to points in the great circle, form a central cone surface. The central cone, with vertex in $\mathbf{O_c}$, projects into the conic $\bar{\mathbf{\Omega}}$ in plane $\mathbf{\Xi}$ (equation 3). Since the image plane and $\mathbf{\Xi}$ are related by collineation $\mathbf{H_c}$, the result of equation 4 comes in a straightforward manner. Notice that conic $\bar{\mathbf{\Omega}}$ degenerate to a line whenever $n_z = 0$.

$$\bar{\mathbf{\Omega}} = \begin{bmatrix} -n_z^2 & 0 & n_x n_z \\ 0 & -n_z^2 & n_y n_z \\ n_x n_z & n_y n_z & n_z^2 \end{bmatrix} \tag{3}$$

$$\hat{\mathbf{\Omega}} = \begin{bmatrix} a & b & d \\ b & c & e \\ d & e & f \end{bmatrix} = \mathbf{H_c}^{-t} \mathbf{\Omega} \mathbf{H_c}^{-1} \tag{4}$$

The catadioptric system is calibrated whenever the collineation $\mathbf{H_c}$ is known. Assume that the image center is $\mathbf{C} = (c_x, c_y)^t$, and that α^2, f_o and s_k are the aspect ratio, the focal length and the skew of the orthographic imaging device. The projective transformation $\mathbf{H_c}$ is given in equation 5 where $f_c = f_o h/2$ is a measurement in pixels of the combined focal length of the camera and the mirror.

$$\mathbf{H_c} = \begin{bmatrix} \alpha f_c & s_k & c_x \\ 0 & \alpha^{-1} f_c & c_y \\ 0 & 0 & 1 \end{bmatrix} \tag{5}$$

Any central catadioptric system can be calibrated from the image of K lines with $K \geqslant 3$. Table 1 summarizes the steps of the calibration method proposed in [3]. The first step is to estimate the conic curves $\hat{\mathbf{\Omega}}_\mathbf{i}$ corresponding to the paracatadioptric projection of K lines in the scene. Any two line images $\hat{\mathbf{\Omega}}_\mathbf{i}$, $\hat{\mathbf{\Omega}}_\mathbf{j}$ intersect in two real points $\hat{\mathbf{B}}_\mathbf{ij}$, $\hat{\mathbf{F}}_\mathbf{ij}$. It can be shown that the image center $\hat{\mathbf{O}}$ must lie in the line $\hat{\mu}_\mathbf{ij}$ going through the two intersection points. Consider the original line in the scene and the plane $\mathbf{\Pi_i}$ going through the effective viewpoint and containing the imaged line (see Fig.

1). If the line is projected in the conic $\hat{\mathbf{\Omega}}_\mathbf{i}$, then the polar line $\hat{\pi}_\mathbf{i}$ of the center $\hat{\mathbf{O}}$ is the image of the vanishing line of $\mathbf{\Pi_i}$. The polar line $\hat{\pi}_\mathbf{i}$ intersects the conic curve $\hat{\mathbf{\Omega}}_\mathbf{i}$ in two points $\hat{\mathbf{I}}_\mathbf{i}$, $\hat{\mathbf{J}}_\mathbf{i}$. It can be proved that these two points lie on the conic $\hat{\mathbf{\Omega}}_\infty$, which is the locus where the absolute conic is mapped by collineation $\mathbf{H_c}$. Conic $\hat{\mathbf{\Omega}}_\infty$ can be estimated using the K pairs of points $\hat{\mathbf{I}}_\mathbf{i}$, $\hat{\mathbf{J}}_\mathbf{i}$. Since $\mathbf{H_c}$ is an upper triangular matrix and $\hat{\mathbf{\Omega}}_\infty = \mathbf{H_c}^{-t}\mathbf{H_c}^{-1}$, then $\mathbf{H_c}$ can be determined from the Cholesky decomposition of $\hat{\mathbf{\Omega}}_\infty$.

The calibration of the paracatadioptric system is straightforward whenever the conic curves corresponding to the line images are known. However the estimation of the these conics using image points is hard to accomplish. There are several algorithms to fit a conic curve to data points [7]. A robust conic fitting algorithm has to cope with noisy data points, biasing due to curvature and partial occlusions. The occlusion problem is of particular importance for our purposes. By occlusion we mean that the available data points lie on a small arc of the curve. It is intuitive that in this circumstances, even for small amounts of noise, it is very hard to obtain the correct conic curve. The present work aims to to cope with this problem using the properties of paracatadioptric line projection .

3 The Line Image $\hat{\omega}$

In general a conic curve has 5 DOF and it can be represented by a symmetric 3×3 matrix $\hat{\mathbf{\Omega}}$ in P^2 (equation 4), or by a point $\hat{\omega} = (a, b, c, d, e, f)^t$ in P^5. From equation 4, replacing $\bar{\mathbf{\Omega}}$, $\mathbf{H_c}$ by the result of equations 3, 5 and assuming $\Upsilon = \alpha s_k c_y - f_c c_x$, yields

$$\begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} -\frac{n_z^2}{\alpha^2 f_c^2} \\ \frac{n_z^2 s_k}{\alpha f_c^3} \\ -\frac{n_z^2}{f_c^2}(\frac{s_k^2}{f_c^2} + \alpha^2) \\ \frac{n_x n_z}{\alpha f_c} - \frac{n_z^2 \Upsilon}{\alpha^2 f_c^3} \\ \frac{\alpha n_y n_z}{f_c} + \frac{\alpha^2 n_z^2 c_y - s_k n_x n_z}{f_c^2} + \frac{s_k n_z^2 \Upsilon}{\alpha f_c^4} \\ \frac{n_z^2 - \alpha c_y n_y n_z}{f_c} - \frac{\alpha^3 c_y^2 n_z^2 - n_x n_z \Upsilon}{\alpha f_c^2} - \frac{n_z^2 \Upsilon^2}{\alpha^2 f_c^4} \end{bmatrix}$$

The paracatadioptric image of a line depends on the intrinsic parameters of the system and on the orientation of the 3D plane $\mathbf{\Pi}$ (Fig. 1). After some algebraic manipulation the previous result can be rewritten in the form of equation 6. If the calibration is known then the conic curve $\hat{\omega}$ is only described by parameters a, d and e. These three parameters encode the scale information and the orientation of plane $\mathbf{\Pi}$ containing the imaged line. Considering that conic $\hat{\omega}$ has 5 DOF, we may conclude that 3 DOF depend on the parabolic system parameters, and the remaining 2 DOF are related with the line that is projected.

$$\hat{\omega} = \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \end{bmatrix} = \begin{bmatrix} a \\ -\frac{\alpha s_k}{f_c} a \\ (\frac{\alpha^2 s_k^2}{f_c^2} + \alpha^4) a \\ d \\ e \\ -\alpha^2 f_c^2 a - c_x d - c_y e \end{bmatrix} \tag{6}$$

4 Paracatadioptric Line Estimation

Assume that we have a paracatadioptric image of K lines in space. Each line is projected in a conic curve $\hat{\omega}_\mathbf{i}$ parameterized has a point in P^5 (equation 7). The goal is to correctly estimate the set of conic curves knowing neither the system calibration nor the position of the lines in the scene.

$$\hat{\omega}_\mathbf{i} = (a_i, b_i, c_i, d_i, e_i, f_i)^t, \ i\text{=1,2,3} \ldots K \tag{7}$$

4.1 Estimation of $\hat{\omega}_\mathbf{i}$ from Image Points

Consider the image points $\hat{\mathbf{x}}^\mathbf{i}_\mathbf{j} = (\hat{x}_j, \hat{y}_j)^t$ with $j = 1, 2 \ldots M_i$ and $M_i \geqslant 5$, lying on conic $\hat{\omega}_\mathbf{i}$. Neglecting the effects of noise it is straightforward that $\mathbf{A_i}\hat{\omega}_\mathbf{i}$ must be null (equation 8).

$$\underbrace{\begin{bmatrix} \hat{x}_1^2 & 2\hat{x}_1\hat{y}_1 & \hat{y}_1^2 & 2\hat{x}_1 & 2\hat{y}_1 & 1 \\ \hat{x}_2^2 & 2\hat{x}_2\hat{y}_2 & \hat{y}_2^2 & 2\hat{x}_2 & 2\hat{y}_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \hat{x}_{M_i}^2 & 2\hat{x}_{M_i}\hat{y}_{M_i} & \hat{y}_{M_i}^2 & 2\hat{x}_{M_i} & 2\hat{y}_{M_i} & 1 \end{bmatrix}}_{\mathbf{A_i}} \hat{\omega}_\mathbf{i} = 0 \tag{8}$$

Equation 9 provides the design matrix $\mathbf{A}$ for the entire set of conic curves $\mathbf{p} = (\hat{\omega}_\mathbf{1}, \hat{\omega}_\mathbf{2} \ldots \hat{\omega}_\mathbf{K})^t$.

$$\underbrace{\begin{bmatrix} \mathbf{A_1} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A_2} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A_3} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A_K} \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} \hat{\omega}_\mathbf{1} \\ \hat{\omega}_\mathbf{2} \\ \hat{\omega}_\mathbf{3} \\ \vdots \\ \hat{\omega}_\mathbf{K} \end{bmatrix}}_{\mathbf{p}} = 0 \tag{9}$$

The equality of equation 9 is only verified in ideal circumstances. In general the data points are corrupted with noise and $\mathbf{Ap} \neq 0$. One way to estimate the set of conic curves is to find the solution $\mathbf{p}$ that minimizes the algebraic distance ϕ (equation 10) under the constraint $\mathbf{p}^t\mathbf{p} = 1$. The minimizer is the normalized eigenvector of $\mathbf{A^t A}$ corresponding to the smallest eigenvalue.

$$\phi(\mathbf{p}) = \mathbf{p}^t \mathbf{A}^t \mathbf{A} \mathbf{p} \tag{10}$$

As referred above the problem is that in general the conic curves corresponding to the paracatadioptric projection of lines are strongly occluded in the image. We are only able to obtain points lying on a small arc of the curve. There are several other conic fitting algorithms which minimizes other distances than the algebraic one [7]. However none of them works properly under these circumstances since the data points do not provide enough information to correctly estimate the conics. This paper proposes to solve the estimation problem by constraining the search space using the properties of paracatadioptric line projection.

4.2 Constraints Verified by a Set of Paracatadioptric Line Images

Assume K lines in the scene that are projected into K conic curves in the paracatadioptric image plane ($K \geqslant 3$). These conic curves can be represented by points of P^5 as shown in equation 7. From the result of equation 6 yields

$$\frac{b_1}{a_1} = \frac{b_2}{a_2} = \frac{b_3}{a_3} = \ldots = \frac{b_K}{a_K} = -\frac{\alpha s_k}{f_c}$$
$$\frac{c_1}{a_1} = \frac{c_2}{a_2} = \frac{c_3}{a_3} = \ldots = \frac{c_K}{a_K} = \frac{\alpha^2 s_k^2}{f_c^2} + \alpha^4$$

From the first expression arises that $\eta_i = 0$ for $i = 2, 3 \ldots K$ with η_i provided by equation 11. Moreover, using the second expression in a similar manner, comes that $\chi_i = 0$ for $i = 2, 3 \ldots K$ where χ_i is given by equation 12.

$$\eta_i = a_1 b_i - a_i b_1, \ i\text{=2,3} \ldots K \quad (11)$$

$$\chi_i = a_1 c_i - a_i c_1, \ i\text{=2,3} \ldots K \quad (12)$$

From equation 6 arises that each line image $\hat{\omega}_\mathbf{i}$ must verify $\alpha^2 f_c^2 a_i + c_x d_i + c_y e_i + f_i = 0$. Consider the conic curves $\hat{\omega}_\mathbf{1}$, $\hat{\omega}_\mathbf{2}$ and $\hat{\omega}_\mathbf{3}$, which are the first three elements of the set of line images. $\alpha^2 f_c^2$, c_x and c_y can be determined as follows

$$\begin{bmatrix} \alpha^2 f_c^2 \\ c_x \\ c_y \end{bmatrix} = -\underbrace{\begin{bmatrix} a_1 & d_1 & e_1 \\ a_2 & d_2 & e_2 \\ a_3 & d_3 & e_3 \end{bmatrix}^{-1}}_{\mathbf{\Phi}} \underbrace{\begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}}_{\mathbf{\Gamma}}$$

If $K > 3$ then each conic curve $\hat{\omega}_\mathbf{i}$ with $i = 4 \ldots K$ must verify the constraint $\nu_i = 0$ (equation 13).

$$\nu_i = \begin{bmatrix} a_i & d_i & e_i & f_i \end{bmatrix} . \begin{bmatrix} -\mathbf{\Phi}^{-1}\mathbf{\Gamma} \\ 1 \end{bmatrix}, \ i\text{=4} \ldots K \quad (13)$$

It is clear that if a set of K conic curves corresponds to the paracatadioptric projection of K lines, then η_i, χ_i and ν_i, provided in equations 11, 12 and 13, must be equal to zero. We have derived $3K - 5$ independent conditions which are necessary for the conic curves to be paracatadioptric line images. However it has not been proved that these conditions are also sufficient. By sufficient we mean that, if a certain set of conic curves verify these conditions then it can be the paracatadioptric projection of a set of lines.

Consider the uncalibrated image of K lines that are mapped in the same number of conics. Since each conic has 5 DOF then a set of K conics has a total of $5K$ DOF. Each line introduces 2 unknowns (DOF), which correspond to the orientation of the associated plane $\mathbf{\Pi}$ (see Fig. 1). Moreover the 5 parameters of matrix $\mathbf{H_c}$ are also unknown (equation 5). Thus there are a total of $2K + 5$ unknowns (DOF). Since $5K > 2K + 5$ then it is obvious that there are sets of conic curves that can never be the paracatadioptric projection of lines. The conics that can correspond to the image of the lines lie in a subspace of dimension $2K + 5$. This means that there are $3K - 5$ independent constraints, which proves the sufficiency of the conditions derived above.

4.3 Estimation of Line Images for Calibration Purposes

Section 4.1 shows how to estimate the set of K paracatadioptric line images using image points. The method consists in finding a solution $\mathbf{p}$ for the conic curves which minimizes the algebraic distance to the data (equation 10). Since the curves appear strongly occluded in the image plane, the estimation results are in general very inaccurate. Section 4.2 shows that a set of K conic curves corresponds to the paracatadioptric projection of K lines, if and only if, it verifies the constraints provided by equations 11, 12 and 13. Our approach consists in using the necessary and sufficient conditions derived above to constraint as much as possible the search space.

4.3.1 General Case

Assume the image of K lines acquired by an uncalibrated paracatadioptric camera. Nothing is known about the parameters of matrix $\mathbf{H_c}$. The skew can be non null and the aspect ratio different from one. Function ϕ provides the algebraic distance between the set of conic curves $\mathbf{p}$ and the data points (equation 10). We aim to minimize of the algebraic distance under the constraints $\eta_i = 0$, $\chi_i = 0$ and $\nu_i = 0$ (equations 11, 12 and 13). One way to achieve this goal is to find the solution $\mathbf{p}$ which minimizes the function ϵ provided in equation 14.

$$\epsilon(\mathbf{p}) = \phi(\mathbf{p}) + \lambda(\sum_{i=2}^{K} \eta_i^2 + \sum_{i=2}^{K} \chi_i^2 + \sum_{i=4}^{K} \nu_i^2) \quad (14)$$

The constraints are introduced as penalty terms weighted by a parameter λ. The minimization of the function ϵ can be stated as a nonlinear least squares problem. The solution can be found using Gauss-Newton or Levenberg-Marquardt algorithms. Notice that the Jacobian matrix can be explicitly derived.

4.3.2 Skewless Images with Known Aspect Ratio

Assume that the orthographic camera is skewless and that the aspect ratio α^2 is known. Replacing s_k by 0 in equation 6 yields $b = 0$ and $c = \alpha^4 a$. The constraints $\eta_i = 0$ and $\chi_i = 0$ for $i = 2 \ldots K$, become $b_i = 0$ and $c_i - \alpha^4 a_i = 0$ for $i = 1 \ldots K$. Notice that there are two additional constraints because now two of the calibration parameters are known. The new function ϵ is given by equation 15.

$$\epsilon(\mathbf{p}) = \phi(\mathbf{p}) + \lambda(\sum_{i=1}^{K} b_i^2 + \sum_{i=1}^{K} (c_i - \alpha^4 a_i)^2 + \sum_{i=4}^{K} \nu_i^2) \quad (15)$$

Making $b_i = 0$ and $c_i = \alpha^4 a_i$ in equation 8 yields

$$\underbrace{\begin{bmatrix} \hat{x}_1^2 + \alpha^4 \hat{y}_1^2 & 2\hat{x}_1 & 2\hat{y}_1 & 1 \\ \hat{x}_2^2 + \alpha^4 \hat{y}_2^2 & 2\hat{x}_2 & 2\hat{y}_2 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ \hat{x}_{M_i}^2 + \alpha^4 \hat{y}_{M_i}^2 & 2\hat{x}_{M_i} & 2\hat{y}_{M_i} & 1 \end{bmatrix}}_{\tilde{\mathbf{A}}_\mathbf{i}} \underbrace{\begin{bmatrix} a_i \\ d_i \\ e_i \\ f_i \end{bmatrix}}_{\tilde{\omega}_\mathbf{i}} = 0$$

A novel design matrix $\tilde{\mathbf{A}}$ can be obtained by replacing $\mathbf{A_i}$ by $\tilde{\mathbf{A}}_\mathbf{i}$ for $i = 1 \ldots K$ in equation 9. Consider $\tilde{\phi}(\tilde{\mathbf{p}}) = \tilde{\mathbf{p}}^\mathbf{t} \tilde{\mathbf{A}}^\mathbf{t} \tilde{\mathbf{A}} \tilde{\mathbf{p}}$ with $\tilde{\mathbf{p}} = (\tilde{\omega}_\mathbf{1}^\mathbf{t} \ldots \tilde{\omega}_\mathbf{K}^\mathbf{t})^t$. The minimum of function $\tilde{\phi}(\tilde{\mathbf{p}})$ is the set of conic curves which minimizes the algebraic distance to the data points and verifies the constraints $b_i = 0$ and $c_i - \alpha^4 a_i = 0$. Thus we can use the objective function $\tilde{\epsilon}$ instead of the one of equation 15

$$\tilde{\epsilon}(\tilde{\mathbf{p}}) = \underbrace{\tilde{\mathbf{p}}^t \tilde{\mathbf{A}}^t \tilde{\mathbf{A}} \tilde{\mathbf{p}}}_{\tilde{\phi}(\tilde{\mathbf{p}})} + \left[\lambda \sum_{i=4}^{K} \nu_i^2 \right] \quad (16)$$

In general the minimization of function $\tilde{\epsilon}$ is still a nonlinear least squares problem. However if $K = 3$ then the second term of equation 16 disappears and the problem becomes closed from. The solution is given by the eigenvector corresponding to the smallest eigenvalue of $\tilde{\mathbf{A}}^\mathbf{t} \tilde{\mathbf{A}}$. Even when $K > 3$ the eigenvector solution is in general quite accurate. In this case the conditions of equation 13 are neglected and the search space is not fully constrained. Nevertheless it is constrained enough to provide good results.

5 Performance Evaluation

Other authors have already proposed algorithms to calibrate a paracatadioptric camera [4, 6, 5]. The approach presented in [6] requires a sequence of paracatadioptric images. The system is calibrated using the consistency of pairwise tracked point features across the sequence, based on the characteristics of catadioptric imaging. In [5], the center and focal length are determined by fitting a circle to the image of the mirror boundary. The method is simple and can be easily automated, however it is not very accurate and requires the visibility of the mirror boundary. Its major drawback is that it is only applicable for the situation of a skewless camera with unitary aspect ratio. Geyer and Daniilidis propose a calibration algorithm using line images [4]. They present a closed-form solution for focal length, image center, and aspect ratio for skewless cameras, and a polynomial root solution in the presence of skew. The line images are estimated taking into account the properties of parabolic projections. Nevertheless the conic curves verifying those properties are not necessarily the paracatadioptric projection of lines. In this section we use simulated images to evaluate the performance of our algorithm and compare it with the one proposed by Geyer and Daniilidis.

5.1 Simulation Scheme

Assume a paracatadioptric camera with a field of view (FOV) of $180°$ and pre-defined intrinsic parameters. The image of a set of K lines is generated as follows. To each line in the scene corresponds a plane $\mathbf{\Pi}$ with normal $\mathbf{n}$ (Fig. 1). The K normals are unitary and randomly chosen from an uniform distribution in the sphere. Each normal defines a plane that intersects the unit sphere in a great circle. Notice that half of the great circle is within the camera field of view. An angle θ, less or equal to the FOV, is chosen to be the amplitude of the arc that is actually visible in the paracatadioptric image. The arc is randomly and uniformly positioned along the part of the great circle which is within the FOV. The visible arc is uniformly sampled by a fixed number N of sample points. The each sample point corresponds a projective ray $\mathbf{x}$. The sample rays are projected using formula 1 and transformed using 2 with the chosen intrinsic parameters. Two dimensional gaussian noise with zero mean and standard deviation σ is added to each image point $\hat{\mathbf{x}}$. As a final remark notice that the amplitude of the visible arc is measured in the great circle where plane $\mathbf{\Pi}$ intersects the sphere, and not in the conic curve where the line is projected. In general the visible angle of the paracatadioptric line image is much less than θ.

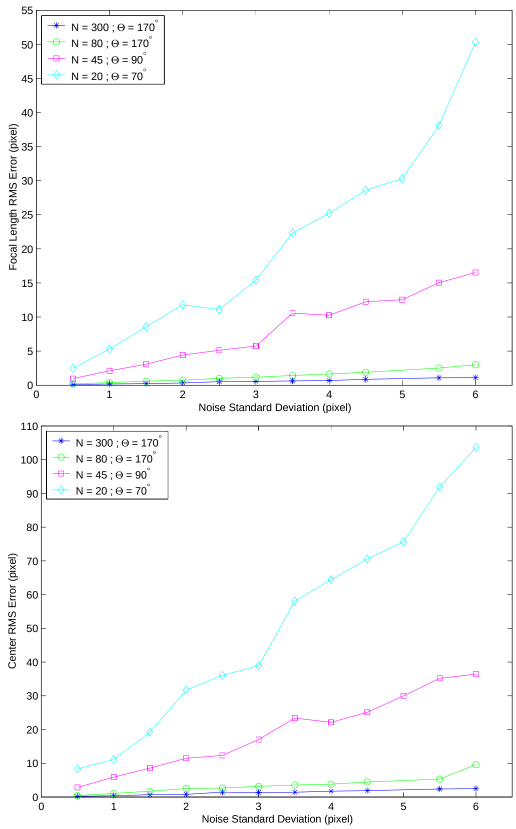

Figure 2: Calibration of skewless system with known aspect ratio using three line images

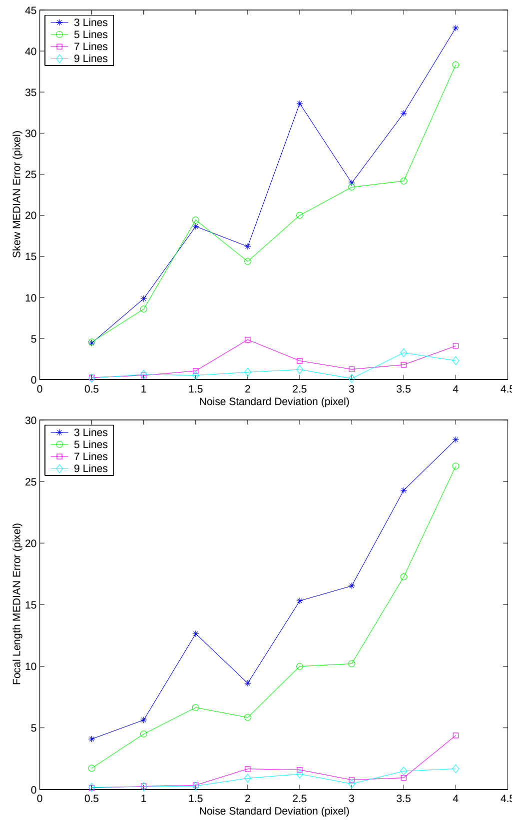

Figure 3: Calibration results for skew and focal length using 3, 5, 7 and 9 lines

5.2 Calibration of Skewless Camera with Known Aspect Ratio

Consider a parabolic camera such that the skew is 0 ($s_k = 0$), the aspect ratio is 1.21 ($\alpha = 1$), and both are assumed to be known. We wish to determine the focal length ($f_c = 245$) and the image center ($(c_x, c_y) = (330, 238)$) using the image of three lines ($K = 3$). The line images are estimated using the closed-form solution proposed at the end of section 4.3.2 and the system is calibrated using the algorithm presented in Tab. 1. The data points are artificially generated using the simulation scheme explained above. The estimated calibration parameters are compared with the ground truth and the RMS error is computed over 100 runs of each experiment.

Fig. 2 shows the results for different choices of θ (amplitude of the visible arc) and N (number of sample points). For each situation the standard deviation of the additive gaussian noise varies between 0.5 and 6 pixels by increments of 0.5 pixels. For $\theta = 170°$ the algorithms presents an excellent performance. The decrease on the number of sample points from 300 to 80 only slightly affects the robustness to noise. Since we are only using three lines, the decrease on the amplitude of the visible arc θ and on the number of points N has a strong impact on the performance. Even so the calibration using arcs of $90°$ is still practicable. The situation of $\theta = 70°$ and $N = 20$ is very extreme leading to a bad estimation of the intrinsic parameters.

The performance of method proposed in [4] is evaluated using similar simulation conditions. A direct comparison can be made between the results presented in here and the ones presented on their paper. In general terms they estimate the conic curves by exploiting the fact that the image center must lie in the line going through the intersection points of any two line images. As discussed in section 4.2, this condition is necessary, but not sufficient, for a set of conic curves to be the paracatadioptric projection of lines. Since the search space is not fully constrained, they need much more than three line images to calibrate the sensor. The results presented in Fig. 2 are obtained using the minimum theoretical number of lines for calibration [3]. Even so, and as far as we are able to judge from the results presented in [4], the performance of our approach seems to be significantly better.

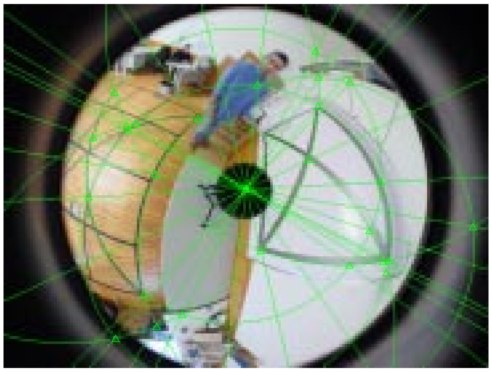

Figure 4: Line images estimation for calibration

		3 Lines	4 Lines	5 Lines	6 Lines
α	*mean*		1.0001	0.9998	0.9996
	std		0.0012	0.0019	0.0015
$\mathbf{f_c}$	*mean*	699.36	699.37	701.03	701.81
	std	17.24	16.00	13.57	10.65
$\mathbf{s_k}$	*mean*		1.46	0.57	-1.95
	std		2.35	1.41	1.39
$\mathbf{c_x}$	*mean*	1137.0	1137.6	1143.6	1147.7
	std	21.4	22.6	11.0	5.8
$\mathbf{c_y}$	*mean*	870.90	870.66	874.36	876.64
	std	11.84	13.42	8.29	5.66

Table 2: Calibrating a paracatadioptric system using K lines

5.3 General Calibration

Consider that nothing is known about the calibration parameters. We wish to determine the aspect ratio, skew, focal length and image center using line images. The amplitude of the visible arc and the number of sample points are respectively $\theta = 140°$ and $N = 140$ (section 5.1). The present experiment compares the performance of the calibration algorithm for 3, 5, 7 and 9 line images. The set of line images is estimated by minimizing the function ϵ provided by equation 14. The solution is found using an iterative gradient descending method. The starting point is the minimizer of the algebraic distance to the data points (equation 10). The calibration parameters are computed (Tab. 1), the results are compared with the ground truth and the median error is computed over 100 runs. The performance of the calibration algorithm can be observed in Fig. 3. As expected the increase in the number of lines improves the robustness of the calibration.

6 Experiments with Real Images

Five images are taken using a paracatadioptric camera with resolution 1704×2272 and the FOV $180°$. Fig. 4 depicts one of the calibration images. For each frame a set of points lying on 6 different line images is selected. The points are used to estimate the conic loci where the line are imaged. Since nothing is known about the sensor calibration, the estimation of the set of line images must be performed by minimizing function ϵ (equation 14). The sensor is calibrated using 3, 4, 5 and 6 lines. Tab. 2 presents the mean and the standard deviation of the calibration results obtained with each image. Notice that the estimated values for the calibration parameters are more or less the same for the different K (number of lines). The standard deviation acts as a measure of confidence. If the results obtained for each image are very different, the variance is high and the achieved calibration is not trustable. As expected the standard deviation decreases when the number of lines increases.

To evaluate the correctness of the calibration we have performed the perspective rectification of a paracatadioptric image of six pairs of parallel lines. The lines were estimated in the rectified image using normal least squares. The angle between each two directions was determined using the corresponding vanishing points and the image of the absolute conic (which is known since the perspective is artificially generated). The angles between each two directions were computed and the results were compared with the angles measured in the scene. The mean of the error was $0.51°$ and the standard deviation was $0.35°$.

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