Consumer's problem

***Note*:** in the model below, P_t^{G2H} is the power from grid to home (G2H) and P_t^{H2G} is the power from home to grid (H2G) in time t (kW). The energy sold to the grid (home to grid) in each time interval is $E_t^{G2H} = P_t^{G2H}\,\Delta t$ and the energy bought is E_t^{H2G}=$P_t^{H2G}\,\Delta t$.

The parameters and the other variables are defined after the model formulation.

$$\min_{P^{G2H},\ P^{H2G}} f = \sum_{i=1}^{I}\sum_{t\in P_i}(x_i\, P_t^{G2H}\,\Delta t) - \sum_{t=1}^{T}\left(c \quad P_t^{H2G}\,\Delta t\right)$$

s. t.

Shiftable loads:

$\sum_{t=T_{1_j}}^{T_{2_j}-d_j+1} s_{jt}^{Sh} = 1 \qquad j = 1 \dots J$

$P_{jt}^{Sh} = \sum_{\substack{r=1 \\ (r\le t\ \wedge\ r\le t+1-T_{1_j})}}^{d_j} f_{jr} \times s_{j(t-r+1)}^{Sh} \qquad j = 1,\dots,J\,, \quad t = T_{1_j},\dots,T_{2_j}$

$P_{jt}^{Sh} = 0 \qquad j = 1,\dots,J\,, \quad t < T_{1_j} \vee t > T_{2_j}$

$s_{jt}^{Sh} \in \{0,1\} \qquad j = 1,\dots,J\,, \quad t = T_{1_j},\dots,T_{2_j} - d_j + 1$

Electric water heater:

$P_t^{losses} = A.U\,(\tau_t - \tau_t^{amb}) \quad , \ t = 1,\dots,T$

$\tau_{t+1} = \left(\frac{M-m_t}{M}\cdot\tau_t + \frac{m_t}{M}\cdot\tau^{net}\right) + \frac{P^R v_t - P_t^{losses}}{M\,c^p}\cdot\Delta t\ , \ t = 0,\dots,T-1$

$\tau_t \ge \tau^{conf}(1 - v_t) \qquad t = 1,\dots,T$

$\tau_t \le \tau^{max} \qquad t = 1,\dots,T$

$P_t^{EWH} = P^R v_t$

$\sum_{t=1}^{T-t^{req}+1} n_t = 1$

$\tau_t \ge \sum_{\substack{t'=1 \\ (t'\le t)}}^{t^{req}} \tau^{req} \times n_{t-t'+1} \qquad t = 1,\dots,T$

$v_t \in \{0,1\}\,, \quad t = 1,\dots,T \qquad ; \qquad n_t \in \{0,1\}\,, \quad t = 1,\dots,T - t^{req} + 1$

Air conditioner:

$\theta_t^{in} = (1-\beta)\,\theta_{t-1}^{in} + \beta\,\theta_{t-1}^{ext} + \gamma\,P^{AC}\,s_{t-1}^{AC}\,, \ t = 1,\dots,T$

$\theta_t^{in} \ge \theta^{min} - \mathcal{M} s_t^{AC} \quad , \qquad t = 1,\dots,T$

$\theta_t^{in} \le \theta^{max} + \mathcal{M}(1 - s_t^{AC}) \qquad t = 1,\dots,T$

$P_t^{AC} = s_t^{AC} P_{AC}^{nom} \qquad t = 1,\dots,T$

$s_t^{AC} \in \{0,1\} \qquad t = 1,\dots,T$

Static battery:

$E_t^B = E_{t-1}^B + (\eta_{ch}^B\,P_t^{H2B}\,\Delta t) - (P_t^{B2H}\,\Delta t\,/\,\eta_{dch}^B) \qquad t = 1,\dots,T$

$E_{min}^B \le E_t^B \le E_{max}^B \qquad t = 1,\dots,T$

$0 \le P_t^{H2B} \le P_{max}^{Bch}\,s_t^{H2B} \qquad t = 1,\dots,T$

$0 \le P_t^{B2H} \le P_{max}^{Bdch}\,s_t^{B2H} \qquad t = 1,\dots,T$

$s_t^{H2B} + s_t^{B2H} \leq 1 \qquad t = 1, \dots, T$

$E_T^B \geq E_0^B$

$s_t^{H2B}, s_t^{B2H} \in \{0,1\} \qquad t = 1, \dots, T$

Electric vehicle battery:

$E_t^V = E_{t-1}^V + (\eta_{ch}^V\, P_t^{H2V}\, \Delta t) - (P_t^{V2H}\, \Delta t \,/\, \eta_{dch}^V) \qquad t = t_a + 1, \dots, t_d$

$E_{min}^V \leq E_t^V \leq E_{max}^V \qquad t = t_a + 1, \dots, t_d$

$0 \leq P_t^{H2V} \leq P_{max}^{Vch}\, s_t^{H2V} \qquad t = t_a + 1, \dots, t_d$

$0 \leq P_t^{V2H} \leq P_{max}^{Vdch}\, s_t^{V2H} \qquad t = t_a + 1, \dots, t_d$

$s_t^{H2V} + s_t^{V2H} \leq 1 \qquad t = t_a + 1, \dots, t_d$

$E_{t_d}^V \geq E_{req}^V$

$s_t^{H2V}, s_t^{V2H} \in \{0,1\} \qquad t = 1, \dots, T$

Power flows:

$0 \leq P_t^{G2H} \leq P^{G\_max}\, s_t^{G2H} \quad , \quad t = 1, \dots, T$

$0 \leq P_t^{H2G} \leq P^{G\_max}\, s_t^{H2G} \quad , \quad t = 1, \dots, T$

$s_t^{G2H} + s_t^{H2G} \leq 1 \quad , \quad t = 1, \dots, T$

$$P_t^{G2H} - P_t^{H2G} + L_t^{PV} = L_t^{Base} + \sum_{j=1}^{J} P_{j,t}^{Sh} + P_t^{EWH} + P_t^{AC} + \left(P_t^{H2B} - P_t^{B2H}\right), \qquad t \leq t_a \vee t > t_d$$

$$P_t^{G2H} - P_t^{H2G} + L_t^{PV} = L_t^{Base} + \sum_{j=1}^{J} P_{j,t}^{Sh} + P_t^{EWH} + P_t^{AC} + \left(P_t^{H2B} - P_t^{B2H}\right) + \left(P_t^{H2V} - P_t^{V2H}\right), t = t_a + 1, \dots, t_d$$

$s_t^{G2H}, s_t^{H2G} \in \{0,1\} \quad , \quad t = 1, \dots, T$

Parameters and variables

Δt: length of the time interval (unit of time) in hours corresponding to the discretization of the planning horizon

Shiftable loads:

Parameters:

J: number of shiftable loads ($j \in \{1, \dots, J\}$)

d_j: duration of load *j* operation cycle

f_{jr}: power requested by load *j* at stage (time) $r = 1, \dots, d_j$ of its working cycle (kW)

$\left[T_{1_j}, T_{2_j}\right] \subset$ T: comfort operation time slot allowed by the consumer for load *j*

Variables:

$s_{jt}^{Sh} = \begin{cases} 1 & \text{if shiftable appliance } j \text{ begins its operation in time } t \\ 0 & \text{otherwise} \end{cases}, t = T_{1_j}, \dots, T_{2_j} - d_j + 1$

P_{jt}^{Sh}: power required by load *j* in time $t \in$ T (kW) (implicit variable determined by s_{jt}^{Sh})

Electric water heater

Parameters:

P^R: power of the resistive heating element (kW)

τ_t^{amb}: ambient temperature around the EWH in time t (ºC), $t = 1, \dots, T$

τ^{net}: inlet water temperature (ºC)

m_t: water withdrawal for consumption in time t (kg), $t = 0, \dots, T$

M: hot water tank capacity (kg)

A: area of the tank envelope (m^2)

U: heat transfer coefficient of the tank (kW/m^2.˚C)

c^p: specific heat of the water (J/kg.˚C)

τ^{max}: maximum allowed temperature (˚C)

τ^{conf}: comfort temperature (˚C)

t^{req}: number of time units required to maintain a certain temperature to eliminate the legionella bacteria

τ^{req}: temperature specified to be kept for t^{req} to eliminate the legionella bacteria (ºC)

Variables (for $t \in$ T)*:*

$v_t \in \{0,1\}$: binary variable defining the off/on control of the heating element in time *t* (v_0 is a constant)

τ_t: hot water temperature inside the tank in time t (ºC), (τ_0 is a constant)

$n_t \in \{0,1\}$: binary variable equal to 1 in the first *t* of the period with duration t^{req} in which $\tau_t > \tau^{req}$ $, t = 1, \dots, T - t^{req} + 1$.

P_t^{losses}: power losses through the envelope in time t (kW), (implicit variables determined by τ_t; P_0^{losses} is a constant)

P_t^{EWH}: power required by the EWH in time *t* (kW), (implicit variables determined by v_t**)**

Air conditioner

Parameters:

$\theta^{min}, \theta^{max}$: minimum and maximum allowed indoor temperature during the planning horizon (ºC)

θ_t^{ext}: outdoor temperature in time *t* (ºC), $t = 0, \dots, T$

P_{AC}^{nom}: nominal power of the AC system (kW)

α, β, γ: coefficients associated with the thermal modeling of the space being conditioned

$\mathcal{M}$: big positive number

Variables (for $t = 1, \dots, T$)*:*

$s_t^{AC} \in \{0,1\}$: binary variable defining the off/on control of the AC system in time *t* (s_0^{AC} is a constant)

θ_t^{in}: indoor temperature in time *t* (ºC), (implicit variables determined by s_t^{AC}; θ_0^{in} is a constant)

P_t^{AC}: power required by the AC in time *t* (kW)

Static battery:

Parameters:

$\eta_{ch}^B, \eta_{dch}^B$: charging and discharging efficiency of the battery

E_{min}^B, E_{max}^B: minimum and maximum allowed battery charge

E_0^B: initial battery charge (kWh)

$P_{max}^{Bch}/P_{max}^{Bdch}$: maximum charge/discharge power allowed for the battery (kW)

$Variables$ (for $t = 1, \dots, T$):

$s_t^{H2B} \in \{0,1\}$: binary variables that are 1 (0) when the battery B is (is not) charging in time t

$s_t^{B2H} \in \{0,1\}$: binary variables that are 1 (0) when the battery B is (is not) discharging in time t

E_t^B: energy in the battery in time t (kWh), (E_0^B is a constant)

P_t^{B2H}: power withdrawn from the battery to home (B2H) in time t (battery discharge)

P_t^{H2B}: power withdrawn from the home to the battery (H2B) in time t (battery charge)

Electric vehicle battery:

Parameters:

t_a, t_d: time of arrival and departure of the EV

$\eta_{ch}^V, \eta_{dch}^V$: charging and discharging efficiency of the EV battery

E_{min}^V, E_{max}^V: minimum and maximum allowed EV battery charge (kWh)

$E_{t_a}^V$: initial EV battery charge, at the time of arrival t_a (kWh)

E_{req}^V: EV battery charge requested at the time of departure t_d (kWh)

$P_{max}^{Vch}, P_{max}^{Vdch}$: maximum charge and discharge power allowed for the EV battery (kW)

$Variables$ (all for $t = t_a + 1, \dots, t_d$):

$s_t^{H2V} \in \{0,1\}$: binary variables that are 1 (0) when the EV battery is (is not) charging in time t

$s_t^{V2H} \in \{0,1\}$: binary variables that are 1 (0) when the EV battery is (is not) discharging in time t

E_t^V: energy in the EV battery in time t ($E_{t_a}^V$ is a constant)

P_t^{H2V}: power withdrawn from the home to the EV battery (H2V) in time t (EV battery charge)

P_t^{V2H}: power withdrawn from the EV battery to home (V2H) in time t (EV battery discharge)

Power flows:

Parameters:

P_{max}^G: maximum power allowed for exchanges with the grid

L_t^{Base}: power of non-controllable base load in time *t* (kW), corresponding to appliances that are not deemed for control (e.g., lighting, fridge, oven, etc.)

L_t^{PV}: local PV power in time t (kWh)

$Variables$ (for $t = 1, \dots, T$)*:*

$s_t^{G2H} \in \{0,1\}$: binary variables that are 1 (0) when the energy is (is not) flowing from grid to home in time t

$s_t^{H2G} \in \{0,1\}$: binary variables that are 1 (0) when the energy is (is not) flowing from home to grid in time t