

# **A Methodology for Reliability Analysis of a Large Multiexchange Network**

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# A Methodology for Reliability Analysis of a Large Multiexchange Network

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## Abstract

Reliability analysis of telecommunication networks in general, and of multiexchange in particular, raise difficult and complex problems in terms of definition, modelling and calculation.

The present work, beyond overviewing related concepts and methods, will present a methodology for the specific problem of reliability analysis of a large circuit-switched multiexchange network. The proposed model takes into account the specific properties of these networks, namely regarding their dimension and uses a multi-parametric network performance evaluation including various grade of service measures. These features required the developments of new algorithms and models for solving some sub-problems. Also current applications and possible developments will be outlined.

**Keywords:** Reliability analysis-performance evaluation; statistical methods; multiexchange networks.

## 1 Introduction

Reliability analysis of telecommunication networks raises difficult and complex problems in terms of definition, modelling and calculation.

Traditionally, till the end of the 70s (see e.g. [13] and [1]), network reliability was approached in terms of network connectivity. This resulted in reliability measures such as the probability that all (or at least some of the nodes) remain connected in the presence of failures in network components. The problems of connectivity analysis and the problems of synthesis of network topologies fulfilling certain connectivity requests has been extensively studied although they usually lead to NP-hard algorithms. This kind of approach was shown to be limited in perspective due to the following reasons: (a) network

components are of high reliability and therefore the probability of physical disconnection between nodes is usually very low; (b) the growing tendency to use multipath routing in communication networks also reduces the probability of functional disconnection between two nodes; (c) network component failures usually lead to some degree of degradation of the network performance, caused by abnormal increase in traffic intensity in parts of the network, thereby provoking eventually unacceptable congestion levels or excessive time delays in message transmission.

This raised the need for reformulating both in conceptual and methodological terms the question of reliability analysis in communication networks. This task was undertaken by several authors, since the 80s, such as Li and Silvester [12], Kubat [9] [10], and Meyer [13], [14], [16]. An essential idea, common to all these approaches, is the combination of reliability analysis with one or several measures of network performance. In this perspective, presently dominant, connectivity measures appear only as one of the factors to be considered in the reliability analysis of communication networks. Nevertheless, in the authors' opinion, connectivity measures still hold a relevant part in the synthesis of network topologies, under certain general traffic routing principles, when a number of requirements has to be guaranteed by the graph representing the topology. Following this line of reasoning were the works in [2], [4] and [6].

A key problem associated with the various approaches to reliability analysis is the enumeration of the possible states of the network, which is particularly critical if the number  $n$  of the failure prone components is high and the calculation of the performance measures for each state of the network has a significant computational cost.

Li and Silvester [12] suggested that only a certain number of states need be considered provided that a large fraction of the space state is covered and developed an algorithm for enumerating such states. Also lower and upper bounds for the selected performance measures were proposed.

Note that the idea of associating, in a probabilistic manner, the states of the system with performance measures is already present in Meyer [13]. In [14], [15] and [16] he developed the concept of "performability", firstly in the context of computing systems and

afterwards in a more broader context of communication networks. The “performability” model of a fault prone system involves the definition of a performance random variable and a model that represents the system and its environment. The performability model is solved when the performance variable distribution is obtained. Note that for large systems, such as is the case of large multiexchange networks, this approach can easily become intractable, since the performance has to be evaluated over all failure scenarios. In [16] a retrospective review of this type of approach is presented. Examples of application of this concept to specific types of telecommunication systems may be seen in [3] and [11].

In [10] it is proposed a general simulation/analytical approach for jointly assessing the reliability and the performance of a communication/computer network. The global states of the network are sampled via simulation and the performance of the network, for each sampled state is obtained analytically. Simulation is therefore used to overcome the state explosion. In [17], it is proposed a new method of reliability analysis that takes into account optimized routing and rerouting after failures.

In the present work a methodology for the specific problem of reliability analysis of a large circuit-switched multiexchange network will be presented. The proposed model, beyond taking into account the specific properties of these networks, namely regarding their dimension, uses a multi-parametric network performance evaluation including various grade of service measures. These features required the developments of new algorithms and models for solving some sub-problems. The necessity of making a distinction in terms of representation of the physical network and of the functional network (and its implications) which seems largely ignored in methodological approaches in this area, is also addressed in the model. A computer-based model for reliability/performance analysis of Lisbon urban network, based on the present methodology, is currently being implemented under a contract of collaboration with TLP (Lisbon & Porto) operator.

In the next section we try to analyse and clarify the underlying basic concepts by referring to the CCITT Rec. E.800. The characterisation of this specific problem of telecommunication network reliability analysis and the description of the proposed method

and the main features of the model are shown in section 3. In section 4 we will briefly present current applications of the methodology and outline further developments.

## 2 Conceptual Framework

Trying to clarify the meaning and relations among several concepts, related to reliability and network performance measures, we will take as a reference the definitions and model of Rec. E.800 of the CCITT. We would like to point out that many of those concepts and their relationships are not always taken exactly with the same meaning, in the literature concerning reliability and quality of service in communication networks. It should also be noted that these concepts and models were developed focusing mainly on public telecommunication networks (e.g. PTNs, ISDNs, PSDNs), and so the authors will report to this kind of networks hereof. Naturally, many of these concepts are applicable or adaptable to computer networks, but the specificity of these networks justifies autonomous treatment (see e.g. [14]).

Let us consider the diagram of Figure 1 which includes the relations between quality of service and network performance, according to Rec. E.800.

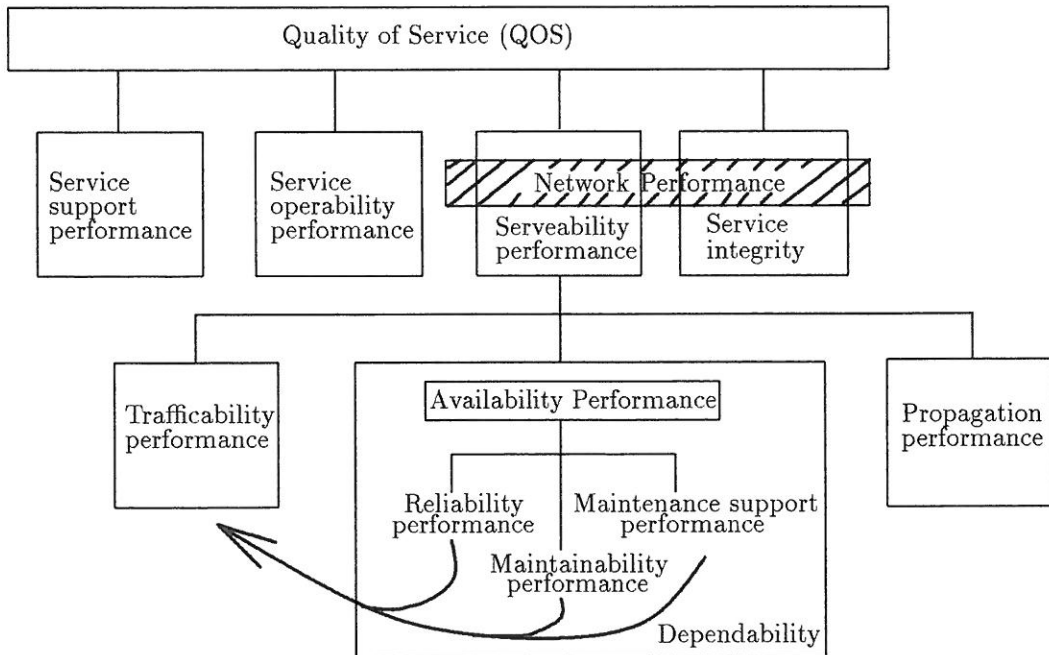


Figure 1: Reliability Analysis and Rec. E.800 model.

The terms in the diagram can be envisaged either as particular levels of quality of service attainable in a certain network, or as project specification requirements or restrictions. The specific contribution of the “organisation” (telecommunications operator, administration) is characterised by the service support concept which defines its capacity to provide a service and support its utilisation. The network (set of technical facilities used in the service, such as transmission systems, switches, controllers, supervision and, in some cases, the terminals) has a global contribution to the quality of service, characterised by the following concepts: service operability, serveability and service integrity [5]. It is obvious that the questions of reliability analysis are situated in the context of the concept of serveability, which is divided into trafficability, dependability and propagation. The performance associated with trafficability is singly expressed in teletraffic engineering terms (see CCITT Rec. E600). The associated performance measures are therefore expressed in loss probabilities (congestions) and mean time delays. The concept of dependability qualitatively represents the combined effect of various aspects: availability, reliability, maintainability and maintenance support, and is related to the capacity of an item, in a certain state, to execute a desired function, for a prescribed period of time. Network performance is, strictly speaking, the capacity of a network, or part of a network, to assure the functions related to the communication between users. The network performance measures, are significant for the operators and quantifiable at the boundaries of the network to which they refer to. Quality of service measures are only quantifiable at a service access point.

In our view the reliability analysis approaches previously mentioned, based in the association of reliability (described in terms of the states of the network components) with network performance measures, try in some way, to elicit the relation between traffic network performance and dependability, expressed through adequate factors. The parameters of network performance (in the sense previously mentioned) intervening in those approaches are associated with measures or characteristics of availability performance of the different components (for example, failure intensities, mean time between failures, stochastic processes describing the failures) and may be expressed in traffic network per-

formance measures (such as call blocking probabilities and mean time delays). It is this interpretation of the paradigm of telecommunication network reliability analysis, which is illustrated by the arrows in Figure 1.

### **3 Approach for a Multiexchange Network**

Next we will present a methodology for the reliability study of multiexchange networks with a significant number of switching centers and functional links, particularly in the context of circuit switched metropolitan networks.

Firstly we will refer to the main characteristics of the addressed problem, and adopted methodological considerations, which lay the foundations of the developed methodology.

#### **3.1 Characterisation of the Problem**

The main objective of the model is the integrated evaluation of the reliability/performance of a large multiexchange network specified in terms of the physical network (including the technical elements which implement the switching and transmission functions) and the associated functional network, described by its switching centers, node to node offered traffic, functional links and routing algorithm.

The model implementation intends to be a flexible tool for assessment of the global effect on network performance of the failures in components of the physical network. Moreover such tool intends to constitute a means of evaluating and comparing alternative network solutions resulting, for example, from alterations in link capacities or in the transmission systems lay-out.

It must be noted that the necessity of considering in the reliability model (concerning at least a significant part of problems of reliability analysis of telecommunication networks) a separate representation of the physical network and of the functional network and its consequences has been largely overlooked in the literature in this area. This simplification, although being reasonable in some applications (an obvious case being the situation where there is a one-to-one correspondence between physical components and functional elements) is clearly unrealistic in the type of problem being addressed in this

work. Also it is unrealistic to assume statistical independency of the failures in functional network, since in many cases, one physical component supports more than one functional element. For example certain physical transmission system may be assigned to various functional links. Moreover the available data regarding component reliability refer to the elements of the physical network, not to the ones of the functional network and although in theory it is possible to calculate the failure probability of the latter as a function of the failure probability of the former, this easily becomes a very difficult and cumbersome task in networks of greater dimension and/or complexity, such as in our case.

In the problem being addressed the functional network is of great dimension: dozens of nodes (digital switching centers) and hundreds of functional links. The physical network has a great number of failure prone elements of different nature. This poses the necessity of reducing according to well defined criteria, the sample state space which has to be considered for the purpose of network performance calculation.

Finally the failure prone components are of great reliability, and from the teletraffic point of view the dominant effects of the most probable failure states are overloads in some parts of the network, increases in call congestions and mean time delays, or in other words a degradation of the grade of service between pairs of exchanges, and associated loss of revenue by the operator regarding telephone, data or other types of traffic carried in the network. Such a network can be considered a gracefully degrading system in the sense defined in [16].

## 3.2 Proposed Methodology

Having in mind the characterisation of the problem under study and the conceptual framework in which it was approached, a methodology of *reliability analysis* with the following main characteristics, was developed:

1. Integration of reliability analysis with functional network performance, translated into an unified approach of parametrisation of the reliability/grade of service, based on the calculation (for each considered physical state of the network components)



of increments of lost traffic between each functional network pair of nodes;

2. Definition of a failure prone component network, based on a description of the physical network structured in components of different technical types and hierarchical levels, and its mapping onto the functional network (see Figure 2). The layers or entities used in the description of the component network include: conducts, coaxial cables and optical fibre cables, microwave links, electro-optical converters, multiplexers, regenerators, line terminating equipment and switching units;
3. Probabilistic description of the failure states in the network, depending on the estimated probabilities of failure of the physical network components;
4. Interactive selection, through an new algorithm [8] (whose main features are outlined in section 3.3), of the most probable states of the component network, depending on the probability coverage of the space state and on the estimated CPU time for the numerical calculations involved in the computation of the performance parameters for each state;
5. Biparametric traffic model that describes all the node to node traffic flows, in all the links of the functional network, that is including a complete description of the traffic flows in every link (marginal means, variances and blocking probabilities). Considering the large dimension of the networks and the use of alternative routing, to achieve greater efficiency in the calculation of node to node blocking probabilities, original numerical algorithms were developed for the traffic sub-models (see some features in [7]);
6. Definition and calculation of a multidimensional set of performance parameters of various types, weighted by the corresponding state probabilities. Such performance measures refer to network performance, both at the level of node to node communication and at the level of global network performance (values averaged over all node pairs) and to link performance, as described in the Appendix. Also the probabilities of certain parameters, namely node to node blocking probabilities

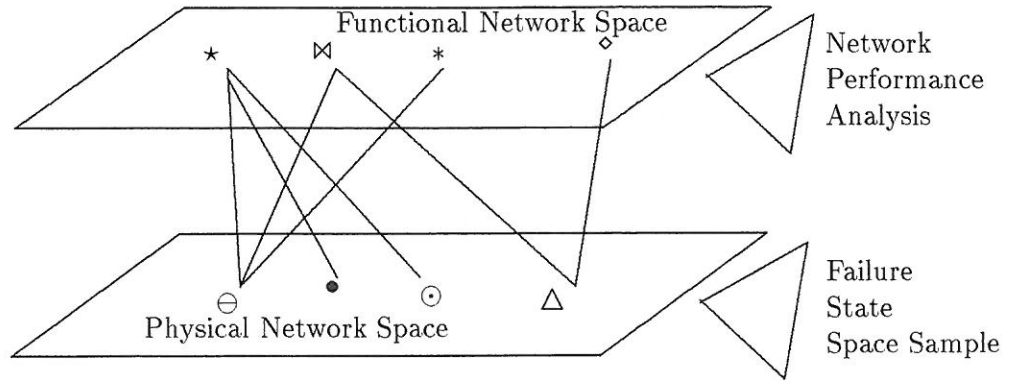


Figure 2: Functional network and physical network within the model.

or average congestions exceeding certain bounds on grade of service, are calculated. That is, in our model, “performability” measures may be defined simultaneously through mean values and through probabilities of exceeding certain performance levels. Moreover an economical assesment of the failure effects is performed through the calculation of the cost associated with the increments in the traffic losses, based on the cost of Erlang.hour for each traffic flow.

The *analysis method* has the following steps:

- (i) Specification of input data representing: (a) functional network, its topology, the node to node offered traffic and the corresponding routing rules; (b) component network, including the failure probability of the components and their mapping on the functional network.
- (ii) Selection of a certain number,  $m$ , of component network states, through an interactive procedure between the computer model and the analyst, involving a compromise among the following factors: (a) the probabilistic characteristics of the failures; (b) some probability coverage of the component network space state; (c) a minimum probability for a significant state; (d) an estimate, even if a rough one, of the numerical processing time associated with the computations for each state.
- (iii) Execution of the performance analysis programme which calculates, given the data and the list of the  $m$  most probable states of the component network, the values

of the selected matrix and scalar parameters. The values of these parameters are calculated state by state and aggregated probabilistically.

- (iv) The analysis can be repeated taking into consideration changes in: (a) the topology of the functional network; (b) routing policies; (c) offered traffic; (d) component characteristics.

### 3.3 The State Generation Algorithm

The specification of a number,  $m$ , of states which guarantee a certain coverage of the state space, is not trivial because it strongly depends on the different failure probabilities of the components in the network.

Li & Silvester [12] proposed an algorithm to obtain the  $m$  most probable states of the network. If those  $m$  states do not guarantee a sufficient state space coverage, a larger value must be given to  $m$  and the whole procedure of state generation has to be restarted. This raised the need for a more flexible state enumeration algorithm as already suggested in [17]. The authors developed and implemented a new algorithm, reported in [8], that (beyond being significantly more efficient than the one in [12]) allows for an interactive and efficient specification of the states, sequentially generated, with decreasing probability, given the failure probability of the components.

The generation process can be temporarily (or definitely) stopped if any of the following conditions is fulfilled:

1. A desired probability coverage of the state space is attained;
2. A maximum number  $m$  of states, considered computationally feasible, given the estimated computational effort of the performance measures (estimated from the approximate average computational cost of obtaining all the performance measures for a state) is attained;
3. The last generated state has a probability less than a minimum value, specified as a significance limit.

The initial values for any of the above mentioned factors, are set at the beginning of the algorithm execution, and when any of them is satisfied, the analyst can decide, based on his/her experience and on the characteristics of the specific network under study, whether the execution of the programme should be stopped or if that factor should be changed to a less restrictive value, allowing for the generation of more states to proceed.

## 4 Applications and Further Developments

The present methodology is being applied to Lisbon urban digital network, under a contract of collaboration between INESC research institute and the TLP public operator.

The functional network under study has dozens of nodes, hundreds of links, and about two to three thousands of parametric equations (depending on the routing being used) in the teletraffic model that enables to calculate the node to node blockings and the marginal traffic flows on the links. As for the component network, it is composed of hundreds of components grouped in several basic types according to their main technical functions and relationship with the functional network.

The authors, in collaboration with TLP engineers involved in the project, intend in the near future, to present and discuss the results of the application of this methodology to this realistic case study.

It is also our intention to address the extension of this type of model to an ISDN network.

## Appendix – Performance measures

From the failure probabilities of the network components, it is possible to calculate the  $m$  most probable states  $S_j$  of the network with  $j = 1, 2, \dots, m$ , according to the algorithm in section 3.3.  $S_1$  is the most probable state: network fully operational and  $S_{2^n}$  is the state with all the components inoperational.

The following parameters are subsequently calculated from appropriate teletraffic models.

## I) Network Performance Parameters

A Node to node blocking probabilities in the fully operational state.

B Average node to node blocking probabilities, for every flow  $f$ :

$$B_{pp}(f) = \sum_{j=1}^m P_j B_{pp}^{(j)}(f)$$

where  $P_j$  is the probability that the network is in state  $S_j$  and  $B_{pp}^{(j)}(f)$  is the node to node blocking for flow  $f$  associated with a pair of nodes, when the network is in state  $S_j$ .

C Node to node lost traffic for every flow  $f$ :

$$L(f) = A(f)B_{pp}(f)$$

where  $A(f)$  is the node to node offered traffic for flow  $f$ .

D Average call congestion in the network for every considered state:

$$B_M = \frac{\sum_f B_{pp}(f)A(f)}{A_T}$$

where  $A_T$  is the total network offered traffic:

$$A_T = \sum_f A(f)$$

E Cost of the average node to node lost traffic for every flow  $f$ , in monetary units:

$$Cost(f) = c(f)L(f)$$

$c(f)$  is the average cost per lost Erlang.hour, expressed in monetary units, which may include the cost associated with telephone traffic and circuit-switched data traffic for flow  $f$ :

$$c(f) = p_d(f)c_d(f) + c_t(f)p_t(f)$$

where  $c_d(f)$  and  $c_t(f)$  are the costs per erlang.hour of lost data and voice traffic (assumed Poissonian) and  $p_d(f)$ ,  $p_t(f)$  are the corresponding fractions with respect to the total traffic  $A(f)$ .

**F** Average network performance in monetary units per Erlang:

$$C_T = \frac{\sum_f Cost(f)}{A_T}$$

**G** Upper and lower bounds [12] for the network average performance in monetary units per Erlang :

$$C_{T_L}(m) = \sum_{j=1}^m P_j D^{(j)} + (1 - \sum_{j=1}^m P_j) D^{(2^n)}$$

$$C_{T_U}(m) = \sum_{j=1}^m P_j D^{(j)} + (1 - \sum_{j=1}^m P_j) D^{(1)}$$

where  $D^{(j)}$  represents the average network performance in state  $S_j$ :

$$D^{(j)} = \frac{\sum_f Bpp^{(j)}(f)A(f)c(f)}{A_T}$$

**H** Number of times a flow disconnection occurs, and respective probability, in a given sample space:

$$N_f = \sum_{j=1}^m I_f^{(j)} \quad ; \quad Prob(\text{disconnection of } f) = \sum_{j=1}^m I_f^{(j)} P_j$$

where

$$I_f^{(j)} = \begin{cases} 1 & \text{if flow } f \text{ is disconnected in } S_j \\ 0 & \text{otherwise} \end{cases}$$

**I** Identification of states for which the node to node blocking probabilities exceed some pre-defined values  $B_k$  (for example  $B_1$  = marginal grade of service tolerable in failure states for brief periods of time, and  $B_2$  = unacceptable grade of service even in faulty conditions, as suggested in [11]) and corresponding probabilities:

$$P_k(f) = \sum_{j=1}^m I_k^{(j)}(f) P_j \quad \text{where} \quad I_k^{(j)} = \begin{cases} 1 & \text{if } Bpp^{(j)}(f) > B_k \text{ in state } S_j \\ 0 & \text{otherwise} \end{cases}$$

**J** Identification of states for which the average call congestion exceeds a certain  $\bar{B}_k$  and corresponding probabilities:

$$\bar{P}_k = \sum_{j=1}^m I_k^{(j)} P_j$$

where

$$I_k^{(j)} = \begin{cases} 1 & \text{if } B_M^{(j)} = \frac{\sum_f Bpp^{(j)}(f)A(f)}{A_T} > \bar{B}_k \text{ in state } S_j \\ 0 & \text{otherwise} \end{cases}$$

**K** Identification of states for which the incremental loss of revenue in the network exceeds some predefined values  $\bar{C}_k$ :

$$P_{c_k} = \sum_{j=1}^m I_{c_k}^{(j)} P_j$$

where

$$I_{c_k}^{(j)} = \begin{cases} 1 & \text{if } \sum_f (Bpp^{(j)}(f) - Bpp^{(1)}(f))A(f)c(f) > \bar{C}_k \text{ in state } S_j \\ 0 & \text{otherwise} \end{cases}$$

The values  $\bar{C}_k$  (for example marginally unacceptable and totally unacceptable loss revenue for the operator) will have to be defined by the analyst usually in terms of a percentage of the expected lost of revenue in the fully operational state.

### I) Link Performance Parameters

**L** Estimate of the mean number of operational circuits in every link  $l$ :

$$\sum_{j=1}^m P_j C_j(l)$$

where  $C_j(l)$  is the number of operational circuits on link  $l$  when the network is in state  $S_j$ .

**M** The increment in the average lost traffic on link  $l$ :

$$\sum_{j=2}^m P_j \sum_{f \text{ that use } l} [B_m^{(j)}(f, l)A_m^{(j)}(f, l) - B_m^{(1)}(f, l)A_m^{(1)}(f, l)]$$

where  $B_m^{(j)}(f, l)$  is the marginal call congestion for flow  $f$  on link  $l$  when the network is in state  $S_j$ . Similarly  $A_m^{(j)}(f, l)$  is the marginal traffic offered by flow  $f$  to a link  $l$  when the network is in state  $S_j$ .

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